

Homework 7
Math 2L03

1. Determine if the following statements are true or False. If true justify your answer. If false, provide a counterexample.
 - (a) If $f'(c) = 0$, then f has a local maximum or minimum at c .
 - (b) If f has a local maximum at c , then f has an absolute maximum at c .
 - (c) If $f(x)$ is an odd function, then $f'(x)$ is an odd function.
 - (d) If $f(x)$ is an even function, then $f'(x)$ is an odd function.
2. Find the critical numbers of the function.
 - (a) $f(x) = x^3 - 12x^2 + 48x$
 - (b) $g(x) = \cos x + \sin x$
 - (c) $h(x) = \frac{x-1}{x^2-x+1}$
3. Find the absolute maximum and absolute minimum of the value of f on the given interval.
 - (a) $f(x) = x^3 - 6x^2 + 9$, $[-3, 3]$
 - (b) $f(x) = \frac{3x-4}{x^2+1}$, $[2, 4]$
4. Find the intervals on which the function is increasing or decreasing, the local maximum and minimum values of the function, intervals of concavity and the inflection points.
 - (a) $f(x) = x^5 - 80x$
 - (b) $g(x) = x^{\frac{1}{3}}(x+4)$
 - (c) $h(x) = \sqrt{x^2+1} - x$
 - (d) $j(x) = \frac{x^2-4}{x^2+4}$
5. Find the
 - Domain
 - Intercepts
 - Symmetry

- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum or Minimum
- Concavity and Points of Inflections

of the following functions and use the information to sketch the curves.

(a) $f(x) = x^3 - 12x + 2$

(b) $g(x) = \frac{x}{\sqrt{x^2 + 1}}$

(c) $h(x) = \frac{1}{x^2 - 9}$

- Find the dimensions of a rectangle with area 400 m^2 whose perimeter is as small as possible.
- If $C(x) = 4000 + 125x - 0.4x^2 + 0.001x^3$ is the cost function and $p(x) = 425 - 1.75x$ is the demand function, find the production level that will maximize profit.
- A right circular cylinder is inscribed in a cone with height 30 cm and radius 18 cm. Find the largest possible volume of such a cylinder.
- The weekly cost to produce x tablets is given by

$$C(x) = 75000 + 100x - 0.03x^2 + 0.000004x^3 \quad 0 \leq x \leq 10000,$$

and the demand function for the tablets is given by

$$p(x) = 200 - 0.05x, \quad 0 \leq x \leq 10000.$$

- Determine the marginal cost, marginal revenue and marginal profit when 250 tablets are sold and when 750 tablets are sold.
- Interpret your answers from part a).
- How many tablets should be produced to maximize profit.